

## Quiz 1 - Group Theory

**1** (4 point) Let  $\mathbb{C}$  denote the set of complex numbers. Define  $*$  on  $\mathbb{C}$  as

$$a * b := a + b + ab \quad \forall a, b \in \mathbb{C}.$$

Is  $(\mathbb{C}, *)$  a group? Justify your answer.

**2** (3 point) Let  $H = \{r \in \mathbb{R} : r^2 \in \mathbb{Q}\}$  where  $\mathbb{R}$  is the set of real numbers and  $\mathbb{Q}$  is the set of rational numbers. Is  $H$  a subgroup of  $\mathbb{R}$ , under addition? Justify your answer.

**3** (3 point) Show that every element in the multiplicative group  $(\mathbb{Z}/8\mathbb{Z})^\times$  has order at most 2. Conclude that  $(\mathbb{Z}/8\mathbb{Z})^\times$  is not cyclic.

### Answers